

DAMTP R/96/24

hep-th/9605010

February 1, 2008

## Heterotic $p$ -branes from Massive Sigma Models

N.D. Lambert<sup>★</sup>*D.A.M.T.P., Silver Street**University of Cambridge**Cambridge, CB3 9EW**England*

### ABSTRACT

We explicitly construct massive  $(0, 4)$  supersymmetric ADHM sigma models which have heterotic  $p$ -brane solitons as their conformal fixed points. These yield the familiar gauge 5-brane and a new 1-brane solution which preserve  $1/2$  and  $1/4$  of the spacetime supersymmetry respectively. We also discuss an analogous construction for the type II NS-NS  $p$ -branes using  $(4, 4)$  supersymmetric models.

---

★ nl10000@damtp.cam.ac.uk

## 1. Introduction

Constructed as Bogomol'nyi solitons of supergravity theories,  $p$ -branes have been studied extensively throughout the history of superstring theory. The extent of their importance is however, only now becoming fully appreciated. The U-duality conjecture [1], along with other string/string and even string/ $p$ -brane dualities, requires that  $p$ -brane soliton states are on an equal footing as string states in the full non-perturbative quantum theory [2]. This interpretation has led to a fundamentally new understanding of string theory in which  $p$ -branes are of central importance.

$p$ -branes are associated with  $(p + 2)$ -form fields strengths tensors (or their 10 dimensional  $(10 - p)$ -form Poincaré duals) of superstring theory. In the type II theories they divide themselves into R-R  $p$ -branes:  $p = 2, 4, 6, 8$  for the IIA string and  $p = 1, 3, 5, 7$  for the IIB string and NS-NS  $p$ -branes:  $p = 1, 5$  for both the IIA and IIB strings. The heterotic strings contain a 1-brane and a 5-brane in the NS sector and the type I string has the same spectrum of  $p$ -branes but in the R-R sector. R-R string states have evaded the standard methods of string theory since they do not couple naturally to the worldsheet, whereas NS (NS-NS) states appear as the moduli of the underlying conformal sigma model. Recently however, Polchinski has provided the R-R  $p$ -brane solitons with a string theory interpretation as spacetime field configurations around a D-brane [3]. Thus the R-R  $p$ -branes can be understood within the language and tools of conformal field theory, with D-branes interpreted as a non-perturbative worldsheet configuration. On the other hand the NS (NS-NS)  $p$ -branes do correspond to exact conformal field theories [4] (indeed we shall construct some of them here) and are therefore much better understood within string perturbation theory. Nevertheless the power of D-branes has led to a wealth of recent results on R-R  $p$ -branes. Furthermore NS-NS  $p$ -branes have also been well studied, in part due to the extended supersymmetry which the type II strings possess. However, heterotic  $p$ -branes have to date received much less attention and this paper may be seen as a small attempt to improve this

situation.

The perturbative massless NS (NS-NS) states of string theory are the moduli of a conformally invariant sigma model. In conformal field theory language they correspond to a truly marginal deformation of the sigma model which does not alter the conformal structure. The addition of a mass term into a conformal sigma model explicitly breaks the conformal invariance. However, we may regain a conformal theory by following the renormalization group flow into the infrared, whereby we move into a new string vacuum - corresponding to deformation by a relevant operator. Here we will consider massive linear supersymmetric sigma models which flow in the infrared to conformal sigma models describing spacetime NS  $p$ -branes. In particular, when the mass is set to zero these models describe ten dimensional Minkowski spacetime, while as the mass tends to infinity (i.e. the infrared limit) they describe the spacetime of an NS (NS-NS)  $p$ -brane.<sup>★</sup>

From the spacetime effective action point of view extremal  $p$ -branes are distinguished as they preserve some supersymmetries of the vacuum and therefore fall into short supermultiplets. As a consequence we expect them to correspond to true states of the full quantum theory. On the worldsheet we need to search for exact conformal field theories. To this end we will look for sigma models with off-shell  $(0,4)$  and  $(4,4)$  supersymmetry as there is a superfield argument that these theories are finite to all orders and therefore yield exact conformal field theories [5,6] (there is a technical problem with anomalies in the chiral models which will be addressed below).

Previously Witten has constructed an  $(0,4)$  supersymmetric massive linear sigma model whose infrared fixed point describes target space ADHM instantons [7,8]. We may tensor the resulting  $(0,4)$  supersymmetric conformal field theory with a flat sigma model on six dimensional Minkowski space to obtain the gauge

---

★ In a sense these massive sigma models may themselves be thought of as soliton-like objects in the space of 2 dimensional quantum field theories since they interpolate between two distinct vacua of string theory as  $m$  ranges from 0 to  $\infty$ .

5-brane of the heterotic string [9,10]. In this paper we will generalize this construction to consider massive linear sigma models whose infrared fixed points are  $(0, 4)$  supersymmetric sigma models on hyper-Kähler target spaces of dimension  $4k, k \in \mathbf{N}$ . For  $k = 1, 2$  we may tensor the resulting theory with a trivial sigma model on flat  $10 - 4k$  dimensional Minkowski space to obtain an exact,  $c = 0$  conformal field theory. Infrared fixed points with  $(0, 4)$  supersymmetry thus naturally lead to a NS 5-brane and 1-brane of the heterotic string, for  $k = 1, 2$  respectively. Similarly conformal fixed points with  $(4, 4)$  supersymmetry correspond to NS-NS  $p$ -branes of the type II strings.

As is well known the gauge (and neutral) 5-brane solution is described by a  $(0, 4)$  supersymmetric sigma model which breaks half of the  $N = 1, D = 10$  supersymmetry of the spacetime effective action [10]. This leads to  $(0, 1)$  supersymmetry of the  $D = 6$  worldsheet effective action [11]. There is a similar picture for the symmetric 5-branes where the sigma model possesses  $(4, 4)$  supersymmetry and thus represents a solution to the type II string as well. Again half of the  $N=2$   $D=10$  spacetime supersymmetry is broken leading to  $(0, 2)$  or  $(1, 1)$  supersymmetry on the  $D = 6$  worldsheet for the type IIA and IIB theories respectively. In the case of a 1-brane there is the well known solution of Dabholkar et al. [12] which also breaks half of the supersymmetry of the  $N=1$   $D=10$  spacetime effective action. Note that as with the symmetric 5-brane, we may view the Dabholkar et al. 1-brane as both a type II and a heterotic string soliton by embedding the gauge connection of the heterotic string into the spin connection. However, because the corresponding sigma model effective action is not in ‘static gauge’, i.e. the metric tangent to the worldsheet is not flat, we cannot conclude that the Dabholkar et al. sigma model effective action admits supersymmetry [13]. Our construction here necessarily considers 1-branes in static gauge and since they are explicitly constructed to admit  $(0, 4)$  supersymmetry it follows that they must preserve at least  $1/4$  of the  $N = 1, D = 10$  spacetime supersymmetry. Furthermore, since this is the maximum amount of supersymmetry a non-trivial sigma model can admit, we cannot expect to find 1-branes in static gauge which preserve more than

1/4 of the spacetime supersymmetry. There also exists in the literature another 1-brane solution, the octonionic string [14]. Like the gauge 5-brane, this solution is unique to the heterotic string. However, the octonionic string soliton preserves one sixteenth of the  $N = 1$   $D = 10$  spacetime supersymmetry and therefore the worldsheet effective action possesses only  $(0, 1)$  supersymmetry.

For the rest of this paper we shall attempt to find NS  $p$ -branes following the above reasoning. In the next section we will construct  $(0, 4)$  supersymmetric massive sigma models and calculate their conformal fixed point. We review the case of a four dimensional target space, already discussed [7,8], leading to the gauge 5-brane. We then discuss the case of an eight dimensional target space which has not been considered before and leads to a new 1-brane solution of the heterotic string. In section three we turn our attention to the type II strings, where an analogous construction with  $(4, 4)$  superconformal fixed points leads to NS-NS  $p$ -branes, although we will not present the details of these models here.

## 2. $(0, 4)$ Supersymmetry

Here we shall use an on-shell  $(0, 4)$  supersymmetric linear sigma model first constructed in [7]. The model consists of  $4k$  bosons  $X^{AY}$ ,  $A = 1, 2$ ,  $Y = 1, 2, \dots, 2k$  with right handed superpartners  $\psi_-^{A'Y}$ ,  $A' = 1, 2$ . There is also a similar multiplet of fields  $\phi^{A'Y'}$ ,  $\chi_-^{AY'}$   $Y' = 1, 2, \dots, 2k'$ . In addition there are  $n$  left handed fermions  $\lambda_+^a$ ,  $a = 1, 2, \dots, n$ . The  $A, B, \dots$  and  $A', B', \dots$  indices are raised (lowered) by the two by two antisymmetric tensor  $\epsilon^{AB}$  ( $\epsilon_{AB}$ ),  $\epsilon^{A'B'}$  ( $\epsilon_{A'B'}$ ). The  $Y, Z, \dots$  and  $Y', Z', \dots$  indices are raised (lowered) by the invariant tensor of  $Sp(k)$ ,  $Sp(k')$  respectively which are denoted by  $\epsilon^{YZ}$  ( $\epsilon_{YZ}$ ),  $\epsilon^{Y'Z'}$  ( $\epsilon_{Y'Z'}$ ).

The mass terms arise from a tensor  $C_{AA'}^a(X, \phi)$ ,

$$C_{AA'}^a = M_{AA'}^a + \epsilon_{AB} N_{A'Y}^a X^{BY} + \epsilon_{A'B'} D_{AY'}^a \phi^{B'Y'} + \epsilon_{AB} \epsilon_{A'B'} E_{YY'}^a X^{BY} \phi^{B'Y'}, \quad (2.1)$$

where  $M_{AA'}^a, N_{A'Y}^a, D_{AY'}^a$  and  $E_{YY'}^a$  are constant tensors, which must satisfy the

constraint

$$C_{AA'}^a C_{BB'}^a + C_{BA'}^a C_{AB'}^a = 0 . \quad (2.2)$$

The action for the theory is given by

$$\begin{aligned} S = \int d^2x \left\{ \epsilon_{AB} \epsilon_{YZ} \partial_{=} X^{AY} \partial_{\neq} X^{BZ} + i \epsilon_{A'B'} \epsilon_{YZ} \psi_-^{A'Y} \partial_{\neq} \psi_-^{B'Z} \right. \\ + \epsilon_{A'B'} \epsilon_{Y'Z'} \partial_{=} \phi^{A'Y'} \partial_{\neq} \phi^{B'Z'} + i \epsilon_{AB} \epsilon_{Y'Z'} \chi_-^{AY'} \partial_{\neq} \chi_-^{BZ'} \\ + i \lambda_+^a \partial_{=} \lambda_+^a - \frac{im}{2} \lambda_+^a \left( \epsilon^{BD} \frac{\partial C_{BB'}^a}{\partial X^{DY}} \psi_-^{B'Y} + \epsilon^{B'D'} \frac{\partial C_{BB'}^a}{\partial \phi^{D'Y'}} \chi_-^{BY'} \right) \\ \left. - \frac{m^2}{8} \epsilon^{AB} \epsilon^{A'B'} C_{AA'}^a C_{BB'}^a \right\} , \quad (2.3) \end{aligned}$$

where

$$\partial_{\neq} = \frac{1}{\sqrt{2}}(\partial_0 + \partial_1) \quad \partial_{=} = \frac{1}{\sqrt{2}}(\partial_0 - \partial_1) ,$$

and  $m$  is an arbitrary mass parameter. This theory then has on-shell (0,4) supersymmetry and, as is discussed by Witten [7] for  $k = 1$ , parallels the ADHM construction of instantons with instanton number  $k'$  in a four dimensional target space. In this paper we will generalise this construction to  $4k$  dimensions.

Here we will consider models for which  $k' = 1$  and  $n = 4k + 4$ . The right handed fermions are  $\lambda_+^a = (\lambda_+^{AY'}, \lambda_+^{Y'Y'})$  and  $C_{AA'}^a$  is chosen to be

$$C_{AA'}^a = B_{AY'}^a \phi_{A'}^{Y'} ,$$

with

$$\begin{aligned} B_{BZ'}^{YY'} &= \epsilon_{B'C'} X_B^Y \delta_{Z'}^{Y'} , \\ B_{BZ'}^{AY'} &= \frac{\rho}{\sqrt{2}} \delta_B^A \delta_{Z'}^{Y'} , \end{aligned} \quad (2.4)$$

where  $\rho$  is an arbitrary positive constant, the size of the  $p$ -brane core, which we will assume is non-zero for now. The bosonic potential for this theory is

$$V = \frac{m^2}{8} (\rho^2 + X^2) \phi^2 , \quad (2.5)$$

where  $X^2 = \epsilon_{AB} \epsilon_{YZ} X^{AY} X^{BZ}$  and similarly for  $\phi^2$ . Thus, for  $\rho \neq 0$ , the vacuum

states of the theory are defined by  $\phi^{A'Y'} = 0$ , and parameterize  $\mathbf{R}^{4k}$ . The  $X^{AY}$  and  $\psi_-^{A'Y}$  are massless fields while  $\phi^{A'Y'}$  and  $\chi_-^{AY'}$  are massive. This yields exactly four of the  $\lambda_+^a$  massive and  $4k$  massless.

To find the conformal fixed point of this model we must first determine which are the massive and massless fields. We then integrate over the massive fields in the path integral and follow the renormalization group flow into the infrared limit. This procedure has been discussed before [8] and so we shall only quote the results here. Due to its  $(0, 4)$  supersymmetry the action (2.3) is ultraviolet finite to all orders of perturbation theory and hence the renormalization group flow is trivial. Furthermore as the massive fields appear only quadratically integrating over them is exact at one loop.

We already know that  $X^{AY}$  and  $\psi_-^{A'Y}$  are massless while  $\phi^{A'Y'}$  and  $\chi_-^{AY'}$  are massive. The  $\lambda_+^a$  we must split up as

$$\lambda_+^a = v_i^a \zeta_+^i + u_I^a \zeta_+^I, \quad (2.6)$$

where we have introduced an orthonormal basis of zero modes of  $B_{AY'}^a$ ,

$$v_i^a B_{AY'}^a = 0 \quad v_i^a v_j^a = \delta_{ij} \quad (2.7)$$

and a similarly defined orthonormal basis for the image of  $B_{AY'}^a$ ,  $u_I^a$ . The massless components of  $\lambda_+^a$  are therefore  $\zeta_+^i$  and the massive components  $\zeta_+^I$ . For the potential given in (2.4) we obtain

$$\begin{aligned} v_{ZZ'}^{YY'} &= \left( \delta_Z^Y - a(X^2) \epsilon_{AB} \epsilon_{CD} X_Z^C X^{DY} \right) \delta_{Z'}^{Y'}, \\ v_{ZZ'}^{AY'} &= \frac{\sqrt{2}}{\sqrt{\rho^2 + X^2}} X_Z^A \delta_{Z'}^{Y'}, \end{aligned} \quad (2.8)$$

where

$$a(X^2) = -\frac{2}{X^2} \left( 1 - \frac{\rho}{\sqrt{\rho^2 + X^2}} \right). \quad (2.9)$$

When  $k = 1$  we may write  $\epsilon_{AB} X_Y^A X_Z^B = \frac{1}{2} X^2 \epsilon_{YZ}$  and these expressions just reduce to those found in [7].

Postponing the problem of chiral and supersymmetry anomalies until later the infrared conformal fixed point is a  $(0, 4)$  supersymmetric sigma model with flat target space  $\mathbf{R}^{4k}$  and Yang-Mills connection

$$A_{ijAX} = v_i^a \frac{\partial v_j^a}{\partial X^{AX}} ,$$

which we obtain to be

$$A_{YY'ZZ'AX} = 2a\epsilon_{Y'Z'}\epsilon_{(Y|X}X_{A|Z)} + a^2\epsilon_{Y'Z'}\epsilon_{CD}X_{A(Y}X_{Z)}^CX_X^D . \quad (2.10)$$

The field strength of this connection can be determined from the standard expression or as the four fermion term in the tree level contribution to the integral over the massive modes [8]. Either way we obtain

$$F_{AYBZ}^{TT'UU'} = -\frac{4}{(\rho^2 + X^2)}\epsilon_{AB}\epsilon^{T'U'} \left( \delta_{(Y}^T + a\epsilon_{CD}X^{CT}X_{(Y}^D \right. \\ \left. \left( \delta_{Z)}^U + a\epsilon_{EF}X^{EU}X_{Z)}^F \right) \right) . \quad (2.11)$$

The classical conformal fixed point action takes the form

$$S = \int d^2x \left\{ \epsilon_{AB}\epsilon_{YZ}\partial_{\neq}X^{AY}\partial_{=}X^{BZ} + i\epsilon_{A'B'}\epsilon_{YZ}\psi_-^{A'Y}\partial_{\neq}\psi_-^{B'Z} \right. \\ \left. + i\zeta_+^{YY'}(\epsilon_{YY'}\epsilon_{ZZ'}\partial_{=}\psi_+^{ZZ'} + A_{YY'ZZ'AX}\partial_{\neq}X^{AX}\zeta_+^{ZZ'}) \right. \\ \left. - \frac{1}{2}\zeta_{+TT'}\zeta_{+UU'}F_{A'YB'Z}^{TT'UU'}\psi_-^{A'Y}\psi_-^{B'Z} \right\} , \quad (2.12)$$

where  $F_{A'YB'Z}^{TT'UU'}$  is simply related to  $F_{AYBZ}^{TT'UU'}$  by replacing  $\epsilon_{AB}$  with  $\epsilon_{A'B'}$  in (2.11).

In [7] Witten shows that for  $k = 1$  the resulting field strength is the same as that obtained using the ADHM construction. Similarly the field strength (2.11) we constructed here corresponds to that obtained using the extended ADHM method [15,20]. Before we move on to discuss the quantum corrections to the



classical action (2.12), it is worth while making some comments on this construction in  $4k$  dimensions. Let us then temporarily switch to a simpler, more familiar, notation. The ADHM construction produces along with the gauge connection  $A_{\alpha\beta i} = v_\alpha^a \partial v_\beta^a / \partial X^i$ , a totally antisymmetric constant tensor  $\eta_{ijkl}$  such that the associated field strength  $F_{ij}^{\alpha\beta}$  satisfies

$$\lambda F_{ij}^{\alpha\beta} = \frac{1}{2} \eta_{ijkl} F_{kl}^{\alpha\beta} . \quad (2.13)$$

Here  $\lambda$  is an eigenvalue which we may take to be 1 by an appropriate redefinition of  $\eta_{ijkl}$ . Now because  $\eta_{ijkl}$  is totally antisymmetric it is easy to see that the standard Yang-Mills equations of motion are satisfied by virtue of the Bianchi identity. Thus for any  $k$  the extended ADHM construction produces a solution of the  $4k$  dimensional Yang-Mills equations which is “self-dual” in the sense of (2.13). In our original notation the self-duality condition (2.13) is simply that (2.11) has the form

$$F_{AYBZ}^{TT'UU'} = \epsilon_{AB} F_{YZ}^{TT'UU'} , \quad (2.14)$$

where  $F_{[YZ]}^{TT'UU'} = 0$ .

In four dimensions there is a unique choice (up to sign) for the tensor  $\eta_{ijkl}$ . In this case it is well known that the ADHM construction produces all possible self-dual field strengths. For  $k > 1$  however there is no natural choice for  $\eta_{ijkl}$ , and certainly no such choice will preserve rotational  $SO(4k)$  invariance. Furthermore the ADHM construction does not produce all the possibilities. The solution we constructed breaks the  $SO(4k)$  rotational invariance of  $\mathbf{R}^{4k}$  to  $SU(2) \times Sp(k)$ . Note that for  $k = 1$ ,  $SU(2) \times Sp(1) \cong SU(2) \times SU(2) \cong SO(4)$  and rotational invariance is preserved. For  $k = 2$  the properties of (2.11) have been studied in [20] where the self-duality equations (2.13) were explicitly written down, although the exact form for  $F$  was not given.

Thus we obtain a non-trivial  $(0, 4)$  supersymmetric sigma model for the conformal fixed point. It is important, however, to check that this model is ultraviolet

finite. While power counting arguments can be constructed to ensure this [5], they rely upon the existence of an off-shell superspace formulation of the theory. The original theory (2.3) only has on-shell supersymmetry but we may promote the supersymmetry of the action (2.12) to off-shell  $(0, 4)$  supersymmetry using constrained superfields [5] since the gauge group is  $Sp(k)$ . (Note from (2.11) that the  $SU(2)$  factor of the  $Sp(k) \times SU(2)$  gauge connection is flat.)

There still remains the problem of anomalies. As is well known the chiral sigma models suffer from anomalies which may be canceled by the inclusion of a non-trivial 3-form  $H$  such that

$$dH = \frac{3}{4}\alpha' \text{Tr} F \wedge F . \quad (2.15)$$

However, there will be additional anomalies as the  $(0, 4)$  supersymmetry is not preserved by the quantization procedure. Indeed there is a two loop divergence for this theory [16] which seems to contradict the power counting arguments. To resolve this requires, in addition to (2.15), that the metric receives a correction in the form of a finite local counter term [6]. This restores the  $(0, 4)$  supersymmetry which is broken by the renormalization procedure and corresponds to the field redefinition

$$g_{AYBZ} = \epsilon_{AB}\epsilon_{YZ} + \alpha' T_{AYBZ} . \quad (2.16)$$

In principle the antisymmetric tensor  $b_{AYBZ}$  may also require a redefinition, however this will not be needed here. Thus the action (2.12), while ultra-violet finite and therefore conformally invariant, requires corrections in the form of finite local counter terms.

Instead of applying the analysis of [6] to eight dimensions we may find the finite local counter term by requiring that the two loop divergences of the action (2.12) are canceled. In this way we obtain, from the general calculation in [16],

(2.16) to be

$$T_{AYBZ} = \frac{12}{(\rho^2 + X^2)} \epsilon_{AB} \epsilon_{YZ} - \frac{12}{(\rho^2 + X^2)^2} \epsilon_{AB} \epsilon_{CD} X_Y^C X_Z^D . \quad (2.17)$$

Such a term presumably also restores the  $(0, 4)$  supersymmetry although we have not explicitly checked this (there is evidence that this must be the case [17]).  $H$  can be simply determined as the Chern-Simons 3-form of the gauge connection (2.10).

In fact (2.17) only ensures the  $\beta$ -functions vanish up to a diffeomorphism  $X^{AY} \rightarrow X^{AY} + \xi^{AY}$  of the worldsheet fields. In particular the metric  $\beta$ -function of the above model is actually

$$\beta_{AYBZ} = -2\alpha' \partial_{(AY} \xi_{BZ)} \quad (2.18)$$

where the vector field  $\xi^{AY}$  is given by the gradient of a scalar,

$$\xi^{AY} = \partial^{AY} \varphi . \quad (2.19)$$

The point of this discussion is that the Curci-Paffuti relations can be used to show that  $\varphi$  is in fact the dilaton [18,19]. To first order in  $\alpha'$  this yields

$$e^{2\varphi} = 1 + 6\alpha' \left( \frac{2k\rho^2 + (2k-1)X^2}{(\rho^2 + X^2)^2} \right) , \quad (2.20)$$

where we have assumed that the dilaton vanishes at infinity.

### The 5-brane

First we consider the  $k = 1$  case corresponding to a four dimensional target space. This case has already been considered in [7,8] but we may find it by simply

setting  $k = 1$  in the above. In this case the gauge group is  $Sp(1) \cong SU(2)$  and we obtain

$$F_{AYBZ}^{TT'UU'} = -\frac{4\rho^2}{(\rho^2 + X^2)^2} \epsilon_{AB} \epsilon^{T'U'} \delta_{(Y}^T \delta_{Z)}^U , \quad (2.21)$$

$$g_{AYBZ} = \left( 1 + 6\alpha' \frac{2\rho^2 + X^2}{(\rho^2 + X^2)^2} \right) \epsilon_{AB} \epsilon_{YZ} , \quad (2.22)$$

$$e^{2\varphi} = 1 + 6\alpha' \left( \frac{2\rho^2 + X^2}{(\rho^2 + X^2)^2} \right) . \quad (2.23)$$

In four dimensions we may write  $H = -\star df$  for some function  $f$ . We then find from (2.15) and (2.21) that

$$\triangle f = 12\alpha' \frac{\rho^4}{(\rho^2 + X^2)^4} ,$$

which can be solved to give

$$H = -\star d\varphi , \quad (2.24)$$

with  $\varphi$  given in (2.23). This is the gauge 5-brane of [9,10] to order  $\alpha'$  and corresponds to the field strength of an instanton on  $\mathbf{R}^4$ . It is completely non-singular for  $\rho > 0$ . The solution was originally constructed as a Bogomol'nyi soliton of the effective supergravity theory breaking half the supersymmetry. The 5-brane has a finite mass per unit 5-volume [9] which saturates the Bogomol'nyi bound. If we let  $\rho \rightarrow 0$  we obtain the neutral 5-brane [10]. The field strength (2.21) vanishes (more precisely it becomes a delta function about  $X^2 = 0$ ). In this case the singularity in the metric at  $X^2$  is moved an infinite spacelike distance away. Thus the solution remains completely non-singular.

### The 1-brane

To find a similar 1-brane solution we consider the case  $k = 2$ , i.e. an eight dimensional target space. The gauge field now lies in  $Sp(2) \cong Spin(5)$ . From

(2.20) the dilaton is

$$e^{2\varphi} = 1 + 6\alpha' \left( \frac{4\rho^2 + 3X^2}{(\rho^2 + X^2)^2} \right) . \quad (2.25)$$

but the field strength (2.11) cannot be simplified in this case. It is instructive then to rewrite the metric in terms of the standard complex coordinates  $\{y, z, t, w\}$  on  $\mathbf{R}^8 \cong \mathbf{C}^4$

$$\begin{aligned} X^{11} &= y & X^{22} &= \bar{y} \\ X^{12} &= z & X^{21} &= -\bar{z} \\ X^{13} &= t & X^{24} &= \bar{t} \\ X^{14} &= w & X^{23} &= -\bar{w} , \end{aligned} \quad (2.26)$$

hence  $X^2 = 2(y\bar{y} + z\bar{z} + t\bar{t} + w\bar{w})$ . The metric is then

$$\begin{aligned} \frac{1}{2}ds^2 &= \left( 1 + \frac{12\alpha'}{(\rho^2 + X^2)} \right) (|dy|^2 + |dz|^2 + |dt|^2 + |dw|^2) \\ &\quad - \frac{12\alpha'}{(\rho^2 + X^2)^2} (|d\alpha|^2 + |d\beta|^2) , \end{aligned} \quad (2.27)$$

where we have introduced the  $Sp(2)$  invariant 1-forms

$$\begin{aligned} d\alpha &= yd\bar{y} + zd\bar{z} + td\bar{t} + wd\bar{w} , \\ d\beta &= ydz - zdy + tdw - wdt . \end{aligned} \quad (2.28)$$

The metric (2.27) is hermitian and positive definite. The torsion  $H_{AXBYCZ}$  can also be found as the Chern-Simons 3-form,

$$H_{AXBYCZ} = \frac{3\alpha'}{2} \text{Tr} \left( A_{[AX} \partial_{BY} A_{CZ]} + 2A_{[AX} A_{BY} A_{CZ]} \right) , \quad (2.29)$$

where the trace is over the  $Sp(2) \times SU(2)$  group indices which we have suppressed in (2.29). The exact form of  $H$  is complicated and so we omit it here.

This solution is only invariant under the group [20]

$$SU(2) \times Sp(2)/\mathbf{Z}_2 \subset SO(8) .$$

While the field strength (2.11) is similar to the octonionic instanton [22,23] the metric (2.27) is clearly different to that of the octonionic 1-brane [14]. In addition the gauge group here is  $Spin(5)$ , whereas the octonionic instanton is specific to an  $SO(7)$  gauge group. A further difference is that the octonionic 1-brane admits only  $(0, 1)$  supersymmetry but our solution explicitly admits  $(0, 4)$  supersymmetry.

We can understand the appearance of these two distinct heterotic 1-branes from the spacetime point of view as follows. In order for the 1-branes to preserve some spacetime supersymmetries it is necessary to have a self-dual field strength [9,10], i.e.  $F_{ij}^{\alpha\beta}$  is an eigenvector of (2.13). This will ensure that the gaugino supersymmetry variation vanishes by projecting the supersymmetry generators  $\Gamma^{ij}\epsilon$  onto a different eigenspace of (2.13). Above four dimensions there are a variety of meanings of “self-duality” corresponding to choices of the tensor  $\eta_{ijkl}$  [20,21]. For example the octonionic instanton also satisfies a set of self-duality equations [22,23] but it cannot be constructed by the ADHM method. This can be seen by noting that the ADHM construction produces three distinct eigenspaces of (2.13) [15,20] (two of which coalesce to  $\lambda = -1$  when  $k = 1$ ), whereas the octonionic instanton has only two [22,23]. Thus as a consequence we may expect a variety of 1-brane solutions of the heterotic string, corresponding to these different notions of “self-duality”.

Although the metric (2.27) is asymptotically flat, its fall off in the eight transverse dimensions is too slow to give the 1-brane a finite ADM mass per unit length. This is a result of the slow fall of  $F$  as  $X^2 \rightarrow \infty$ . Since all solutions to the Yang-Mills equations have infinite action in dimensions greater than four presumably all such 1-branes have infinite masses per unit length, as is the case of the octonionic 1-brane. It is impossible to say whether or not the solution saturates a Bogomol’nyi bound since  $H$  falls off only as fast as  $1/X^3$  at transverse infinity and therefore the Bogomol’nyi bound also diverges [9,10,14].

In either of the two complex planes  $\{y, z, 0, 0\}$  and  $\{0, 0, t, w\}$  the metric (2.27) reduces to the 5-brane metric (2.22) and the field strength (2.11) to that of a four dimensional instanton. This suggests, along with the fact that only  $1/4$  of the spacetime supersymmetry is preserved, that this solution does not represent a new solitonic string, but rather some kind of bound state of two gauge 5-branes, intersecting in 1-brane. Viewed as intersecting 5-branes the infinite mass per unit length arises because the entire energy density of the two 5-branes has been squeezed into the one dimensional intersection.

For  $\rho > 0$  the solution is completely non-singular. In the limiting case  $\rho \rightarrow 0$  we find that the field strength tensor does not vanish and in fact diverges at  $X^2 = 0$ , although metric again moves this point an infinite spacelike distance away. If we let  $\rho \rightarrow 0$  in the gauge 5-brane we obtain the neutral 5-brane and  $H$  becomes a closed 3-form (the Yang-Mills field strength vanishes). As a result of this we may find a  $(4, 4)$  supersymmetric solution by simply setting  $A_{AX} = \omega_{AX}^{(-)}$ , rather than  $A_{AX} = 0$ . This yields the symmetric 5-brane [10]. It is reasonable to ask if there is a similarly related  $(4, 4)$  supersymmetric solution to the  $(0, 4)$  supersymmetric 1-brane. However, since the Yang-Mills field is non-zero for all  $\rho$  in (2.11),  $H$  is never closed and hence this solution has no analogue of the neutral 5-brane. This is also the case for the Octonionic 1-brane. We now consider a brief discussion of what the conformal fixed points with  $(4, 4)$  supersymmetry should be.

### 3. $(4, 4)$ Supersymmetry

In order to apply an analogous analysis to the type II strings we must find corresponding massive sigma models whose conformal fixed points admit  $(4, 4)$  supersymmetry. The analysis of these theories is significantly different to those above as there are no anomalies and no extraneous two loop divergence which need to be canceled. Indeed it may be that  $(4, 4)$  supersymmetry is too restrictive to produce a non-trivial conformal sigma model in the infrared limit. We do not address this issue here but instead we will try to deduce the form of the conformal

fixed point from general considerations. Similar discussions have appeared in [24], where fixed points of the model (2.3) with  $(4, 4)$  supersymmetry were studied, and also in [25] for  $k = 1$  where general conformal field theory arguments were employed.

The conditions on the potential of a linear  $(4, 4)$  sigma model have been discussed in [26,27]. With the same bosonic fields as the  $(0, 4)$  supersymmetric case the potential has the form

$$V = \frac{1}{2}m^2 X^2 \phi^2 . \quad (3.1)$$

As before there is a rigid  $SU(2) \times Sp(k)$  symmetry of the theory. The classical vacua lie at either  $\phi^{A'Y'} = 0$  or  $X^{AY} = 0$  and in the  $k = 1$  case there is a symmetry between the two branches. We choose the  $4k$  dimensional vacua where  $\phi^{A'Y'} = 0$ . The theory then has four massive fields  $\phi^{A'Y'}$  and their fermionic partners and  $4k$  massless fields  $X^{AY}$  with their fermionic partners. The other vacuum, where  $X^{AY} = 0$ , is four dimensional with  $4k$  massive bosonic fields and their fermionic partners. Both branches of the classical vacuum moduli space ‘touch’ and form a singularity at the origin  $\phi^{A'Y'} = X^{AY} = 0$ , where the manifold structure degenerates.

It is not hard to convince oneself that since there is no need for an equation of the form (2.6), classically the resulting conformal field theory is a trivial linear sigma model. However, we must still consider the quantum corrections which are again exact at one loop since (3.1) is quadratic in the massive fields. These corrections may be expected since there is a singularity in the vacuum moduli space at  $X^2 = 0$ , where the fields we integrated over degenerate to zero mass. The possible corrections to the trivial classical effective action at one loop come in the form of finite local counter terms for the metric and torsion  $g_{AYBZ}$  and  $H_{CTAYBZ}$ , respectively. Following [16] the counter term  $T_{AYBZ}$  for  $g_{AYBZ}$  must satisfy

$$\partial^2 T_{AYBZ} = 0 . \quad (3.2)$$

The general form  $T_{AYBZ}$  which satisfies (3.2) and is invariant under  $SU(2) \times Sp(k)$



is

$$T_{AYBZ} = \left( \frac{Q}{X^{4k-2}} + \frac{Q'}{X^{4k}} \right) \epsilon_{AB} \epsilon_{YZ} - 2k \frac{Q'}{X^{4k+2}} \epsilon_{AB} \epsilon_{CD} X^C_Y X^D_Z , \quad (3.3)$$

where  $Q$  and  $Q'$  are constants. Using the same method as above we then obtain the dilaton to be

$$e^{2\varphi} = 1 + \frac{Q\alpha'}{X^{4k-2}} , \quad (3.4)$$

where we have assumed that  $\varphi$  vanishes at infinity. Given this form for  $T_{AYBZ}$  the torsion counter term  $X$  must satisfy

$$\partial^{CT} X_{CTAYBZ} = 0 . \quad (3.5)$$

These conditions only ensure that the sigma model remains finite at two loops [16]. This is required by, but is not by itself sufficient for,  $(4, 4)$  supersymmetry. In addition  $g_{AYBZ}$  and the generalized connection  $\omega_{AY}^{(-)}$  must be compatible with the hyper-Kähler structure [5]. One can explicitly check that for these target space fields the sigma model  $\beta$ -functions vanish to order  $\alpha'$ , i.e.  $R_{AYBZ}^{(-)} = \mathcal{O}(\alpha'^2)$ . We will assume from our discussion that there is a choice for  $X$  which preserves the  $(4, 4)$  supersymmetry. In this case the generalised curvature tensor with torsion will satisfy the self-duality condition, analogous to (2.14),

$$R_{AYBZ}^{(-)}{}_{CTDU} = \epsilon_{AB} R_{YZ}^{(-)}{}_{CTDU} \quad (3.6)$$

with  $R_{[YZ]}^{(-)}{}_{CTDU} = 0$  and the sigma model  $\beta$ -function will vanish to all orders in  $\alpha'$ .

First let us consider the case  $k = 1$ , where the target space of the conformal fixed point is four dimensional. Here there is a simple form for  $H$ , as used above

in the  $(0, 4)$  supersymmetric case, and we obtain the metric, dilaton and torsion

$$\begin{aligned} g_{ABYZ} &= \left(1 + \frac{Q\alpha'}{X^2}\right) \epsilon_{AB}\epsilon_{YZ} , \\ e^{2\varphi} &= 1 + \frac{Q\alpha'}{X^2} , \\ H &= - \star d\varphi . \end{aligned} \tag{3.7}$$

This is easily recognizable as the symmetric 5-brane of [10] and is indeed an exact  $(4, 4)$  superconformal field theory with the curvature satisfying (3.6) [24].

We now turn to the  $k = 2$  case. The metric, written in the same coordinates as (2.27), and dilaton now take the form

$$\begin{aligned} \frac{1}{2}ds^2 &= \left(1 + \frac{Q\alpha'}{X^6} + \frac{Q'\alpha'}{X^8}\right) (|dy|^2 + |dz|^2 + |dt|^2 + |dw|^2) \\ &\quad - 2\frac{Q'\alpha'}{X^{10}}(|d\alpha|^2 + |d\beta|^2) . \\ e^{2\varphi} &= 1 + \frac{Q\alpha'}{X^6} , \end{aligned} \tag{3.8}$$

Unfortunately however, our simple analysis does not provide a form for  $H$ . To remedy this a more detailed argument using the harmonic superfield analysis of [24] would be useful.

## 4. Comments

In this paper we have explicitly given massive  $(0, 4)$  supersymmetric sigma models whose conformal fixed points are NS  $p$ -branes of the heterotic string and computed the leading order term required to cancel the anomalies. We have also considered the case of the type II strings but not pursued the details here. We hope to fully address the  $(4, 4)$  supersymmetric sigma models in a future work. In conclusion we would like to make some comments.

As was discussed in [25,8] for the  $k = 1$   $(0, 4)$  model in section two, taking the limit  $\rho \rightarrow 0$  after the conformal fixed point is found yields a non-trivial spacetime, identical to (2.24). On the other hand, if we start with  $\rho = 0$  in (2.3), integrating over the massive  $\phi^{A'Y'}$  modes apparently does not provide any quantum corrections since the connection is flat and the anomalies vanish. There is still a singularity at  $X^2 = 0$  where the  $\phi^{A'Y'}$  fields degenerate to zero mass. We would then be led to the false conclusion that the vacuum moduli space is flat with a degenerating manifold structure at the origin. But this is exactly the same situation that appears with a  $(4, 4)$  theory based on the potential (3.1). This lends further support to the conjecture that (3.7) and (3.8) with  $Q, Q' \neq 0$  are indeed the  $(4, 4)$  superconformal fixed points.

Another generalization of the above analysis would be to add mass terms to an exact conformal sigma model with a curved target space and then similarly obtain the infrared fixed point conformal field theory. For example the linear model considered here could be modified to a massless  $(4, 4)$  supersymmetric sigma model with a curved target space and mass terms which then break the supersymmetry to  $(0, 4)$ . In particular one could consider the sigma model of the symmetric 5-brane [10]. Since this metric is conformally flat and the Yang-Mills equations conformally invariant in four dimensions, the ADHM construction can again be applied.

In the  $k = 1$  cases above it is also possible to tensor the infrared fixed point theory with another  $(0, 4)$  or  $(4, 4)$  conformal field theory on a compact hyper-Kähler four manifold  $K$  (for example  $K = \mathbf{T}^4$  or  $K_3$ ) and then again with a trivial sigma model on two dimensional Minkowski space. With this construction we obtain a 1-brane in string theory compactified to six dimensions on  $K$ , but with the metric and 3-form (3.7) in the transverse space. This is the construction of [4] and from the ten dimensional point of view corresponds to a 5-brane wrapped around  $K$ . These solutions preserve 1/2 of the  $D = 6$  spacetime supersymmetry and hence 1/2 or 1/4 of the  $D = 10$  spacetime supersymmetry for  $K = \mathbf{T}^4, K_3$  respectively.

Another possibility for the  $k = 1$  cases is to tensor the conformal fixed point theory with another copy of itself and then with a trivial two dimensional sigma model. Thus we arrive at the double instanton solution [29] in the  $(4, 4)$  supersymmetric case and a similar heterotic version in the  $(0, 4)$  case. As one may expect these solutions preserve  $1/4$  of the spacetime supersymmetry. The double instanton 1-branes have the property that the metric singularity is located at the origin of coordinates in either of the four dimensional target spaces and not at a single point in the transverse space. They are therefore also most appropriately viewed as two orthogonal intersecting 5-branes in ten dimensional spacetime [30], although in this case the metric and torsion are singular not just on the intersection of the two 5-branes but also on the world sheets of each. Again these 1-branes have infinite masses per unit length but this may be resolved by viewing them as intersecting 5-branes with the energy density spread out over the two 5-branes.

Recently there has been interest in the intersecting  $p$ -branes of M theory [31,32,30]. Here we have seen that the situation is somewhat different for heterotic strings as the 1-brane solutions discussed here do not depend on harmonic functions in the transverse space, because the non-vanishing Yang-Mills field strength acts as a source for Laplace's equation. In the future it would be interesting to understand why both the double instanton and the solution found here both occur as intersecting heterotic 5-branes and if the octonionic string can be viewed in a similar light.

I would like to thank G. Papadopoulos and P.K. Townsend for discussions.

## REFERENCES

1. C.M. Hull and P.K. Townsend, Nucl. Phys. **B438** (1995) 109
2. P.K. Townsend, “ $p$ -brane Democracy”, in the proceedings of the PASCOS conference, March, 1995, hep-th/9507048
3. J. Polchinski, Phys. Rev. Lett. **75** (1995) 4724
4. D. Kutasov, “Orbifolds and Solitons”, EFI-95-79, hep-th/9512145
5. P. Howe and G. Papadopoulos, Nucl. Phys. **B289** (1987) 264
6. P. Howe and G. Papadopoulos, Nucl. Phys. **B381** (1992) 360
7. E. Witten, J. Geom. Phys. **15** (1995) 215, hep-th/9410052
8. N.D. Lambert, Nucl. Phys. **B460** (1996) 221, hep-th/9508039
9. A. Strominger, Nucl. Phys. **B343** (1990) 167
10. C.G. Callan Jr., J.A. Harvey and A. Strominger, Nucl. Phys. **B359** (1991) 611
11. C.G. Callan Jr., J.A. Harvey and A. Strominger, Nucl. Phys. **B367** (1991) 60
12. A. Dabholkar, G. Gibbons, J. Harvey and F. Ruiz-Ruiz, Nucl. Phys. **B340** (1990) 33
13. J. Hughes and J. Polchinski, Nucl. Phys. **B278** (1986) 147
14. J. Harvey and A. Strominger, Phys. Rev. Lett. **66** (1991) 549
15. E. Corrigan, P. Goddard and A. Kent, Comm. Math. Phys. **100** (1985) 1

16. N.D. Lambert, Nucl. Phys. **B469** (1996) 68, hep-th/9510130
17. P. Howe and G. Papadopoulos, “Twistor Spaces for HKT Manifolds” hep-th/9602108
18. G. Curci and G. Paffuti, Nucl. Phys. **B286** (1987) 399
19. I. Jack, D.R.T. Jones and D.A. Ross, Nucl. Phys. **B307** (1988) 531
20. R.S. Ward, Nucl. Phys. **B236** (1984) 381
21. E. Corrigan, C. Devchand, D.B. Fairlie and J. Nuyts, Nucl. Phys. **B214** (1983) 452
22. D.B. Fairlie and J. Nuyts, J. Phys. **A 17** (1984) 2867
23. S. Fubini and H. Nicolai, Phys. Lett. **155B** (1985) 369
24. A. Galperin and E. Sokatchev, Class. Quan. Grav. **13** (1996) 161
25. E. Witten, “Some Comments on String Dynamics”, in the proceedings of *Stings '95*, USC, March 1995, hep-th/9507121
26. L. Alvarez-Gaumé and D. Freedman, Comm. Math. Phys. **91** (1983) 87
27. G. Papadopoulos and P.K. Townsend, Class. Quan. Grav. **11** (1994) 515
28. G. Horowitz and A.A. Tseytlin, Phys. Rev. **D51** (1995) 2896
29. R.R. Khuri, Phys. Rev. **D 48** (1992) 2947
30. J. Gauntlett, D. Kastor and J. Traschen, “Overlapping Branes in M- Theor”, hep-th/9604179
31. G. Papadopoulos and P.K. Townsend, “Intersecting M-branes”, hep-th/9603087

32. A.A. Tseytlin, “Harmonic Superpositions of M-branes”, hep-th/9604035